JULIA LOUGOVAYA

Dissemination of Mathematical Knowledge and Skills in Late Antique Egypt

Editing Mathematical Problems Preserved in Papyrus Codices

Keywords late antiquity; critical edition; Greek Papyrus codices; History of Science and Education; mathematical problems

Project Participant
Julia Lougovaya
Institutional Affiliation
Heidelberg University: Institute for Papyrology
Term of the Project
2023–2026

Abstract of the Project
The project "Dissemination of Mathematical Knowledge and Skills in Late Antique Egypt" focuses on mathematical education of 'practitioners'—a wide range of people for whom computational and metrological skills were part of their daily life and work in late antique Egypt. Its core is formed by an edition of several Greek papyrus codices and fragments thereof which contain collections of mathematical problems, metrological texts and arithmetical tables (see, for example, Fig. 1). All dating to the 4th through 5th centuries, these codices reflect a practice of purposeful collecting, selecting and copying of mathematical and metrological texts, which circulated earlier usually in the form of rolls and likely also transmitted orally in a variety of contexts.
Editing texts of practical mathematics preserved in papyri is challenging for a variety of reasons. As usual for texts found in excavations, most are preserved only fragmentarily. But an even bigger difficulty lies in the concise style of mathematical texts in papyri. A large part of education in antiquity in general and of mathematical education in particular took place orally. Consequently, problems and their solutions recorded in papyrus codices were meant to supply just enough information for the reader to be able to expound orally both the problem and the procedures used to solve it. As a result, these texts omit words, use incomplete sentences, lack syntactic coordination, leave out information or logical steps that may have seemed obvious to their composers or copyists, and employ multiple and varying abbreviations. Moreover, collections of problems were produced by and for practitioners who had limited access to basic education. This means that writers of these codices were easily prone to making mistakes of various kinds, such as in

 Erschienen in: Isabel Langkabel, Ludger Lieb, Maximiliane Nietzschmann and Lena Sowada (Hgg.), *Editionswissenschaft – Textkritik – Digital Humanities. 25 Heidelberger Editionsprojekte,* Heidelberg 2026, 19–25. DOI: https://doi.org/10.17885/heiup.1634.c23580

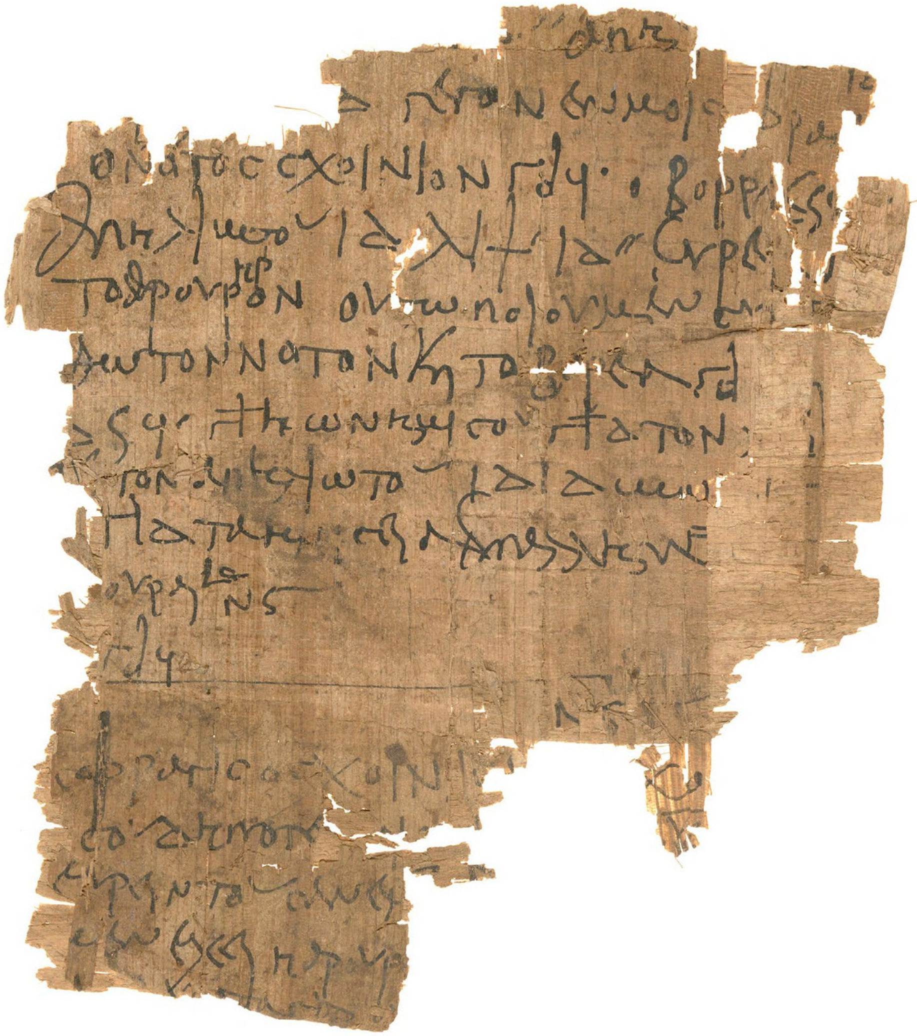

Fig. 1: A codex page inscribed with mathematical problems (P.Col. inv. 157a).

spelling, calculation, misunderstanding of copied text or diagrams, and misuse of terms. How these mistakes are treated in an edition presents a further challenge in dealing with mathematical texts. Finally, there is the challenge of not converting mathematics of the papyri to the language of modern algebraic notation, a common anachronistic approach that might make the problems easier for a modern reader to follow, but at the price of failing to understand ancients' perception of the problems and their methods to solve them.

A Mathematical Problem from a Papyrus Codex

Provenance unknown 12 (h) × 10.2 (w) cm Fourth century
P.Col. inv. 157a
Ed. pr.: Bakker 2007.

Side A ↓
No. 1
Problem: Given area *A* and side *a* of a quadrilateral, find out the other three sides *b*, *c* and *d*.
Procedure: $(A, a) \rightarrow A = e \times f.\ \ e \times 2 - a = b, f = c = d.$

[ ] ̣ ̣ // ἀπηλι̣[ώ]τ̣ο̣[υ] ιδ̣
κ̣α̣ὶ̣ λ̣[ὶψ ι]δ. γέγονέν μοι σφραγί̣[ς˙]
ὁ νότος σχοινίον γdη, ὁ βορρ(ᾶς) δ̣∫η̣,
ἀπηλιώτου ιδ, λὶψ ιδ.// εὑρεῖ̣ν
τὸ ἀρουρ\ηρ/όν. οὕτω ποιοῦμεν. συ̣ν̣θ̣[ί-]
θω τὸν νότον καὶ τὸ βορέα, γd[η κ]α̣ὶ
δ∫η˙ γί(νεται) η˙ ὧν ἥμισου˙ γί(νεται) δ. τὸν λ[ί]β̣[α]
κ̣[αὶ] τὸν ἀπηλιώτου, ιδ ιδ, ὧν ἥ̣μ̣[ι]σ̣[ου˙]
[γ]ί(νεται) ιδ. τὰ ἡμίσεια ἐπ’ ἄλληλα γ[ί](νεται) [νς]
[ἄρ]ουραι\ ̣ ̣/ νς vacat

diagram γdη ______________________________
νς

1 [ ] ̣ [̣] ̣ // ed.pr. 3 l. σχοινίων | γdη˙ pap | βορρ(έας) ed.pr. | δ̣∫η̣˙ pap 4 λιψ´ pap 5 l. ἀρουρηδόν: α *corr. ex* υ 5 l. συντίθω 6 l. τὸν βορέαν 7 δ∫η˙ pap 7, 8 l. ἥμισυ 8 l. ἀπηλιώτην 9 l. ἡμίσεα

Translation

... // 14 on the east and 14 on the west. I have got a plot 3 1/4 1/8 schoinia on the south, 4 1/2 1/8 on the north, 14 on the east, 14 on the west 14. // To find out the area. We do it this way: I add the south and the north, 3 1/4 1/8 and 4 1/2 1/8, it becomes 8, half of which is 4; the west and the east, 14 and 14, half of which is 14. Multiply the halves by each other, it becomes 56 arourai.

diagram 3 1/4 1/8 ______________________________

56

Notes

1 The traces before the double slash might be compatible with ι̣δ̣.

2 The restoration κ̣α̣ὶ̣ λ̣[ὶψ ι]δ is tentative and based on line 4. The word may have been in the genitive, λ[ιβὸς ι]δ. Here and elsewhere in the text I do not regularize cases of the cardinal points unless they are introduced by an article. In those cases, the apparatus provides a form that agrees with the article.

10 It is difficult to understand what is written in the insertion (Fig. 2). It looks like a symbol featuring an alpha, and, if so, one wonders if it could be a symbol for arourai here.

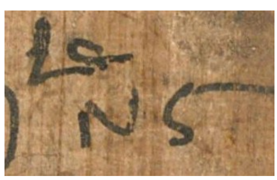

Fig. 2: Detail of the mathematical problem depicted in Fig. 1.

diagram: What is printed in the edition as [(. .)] γdη *vacat* in line 11 and *vacat* νς̣ in line 12, separated by a long line, all belongs to a diagram illustrating the problem, in which the horizontal line represents the plot. Such diagrams, with dimensions written at left, right, on top and below a horizontal line are well attested in land surveys preserved in papyri. For a general explanation, see Kenyon's introduction to P.Lond. II 267 (2nd c., Arsinoite) and cf. Fowler 1999, 231–234, for a detailed analysis of computations carried out in such surveys on the example of O. Bodl. II 1847 (30 BCE–14 CE, Thebes); for further numerous examples of representing a plot of land by a line with the four dimensions written around it, see, for example P.Tebt. I 87 (116–115 BCE, Arsinoite) with images at Papyri.info (https://papyri.info/ddbdp/p.tebt;1;87/images), and for demotic evidence, cf. Friberg 2005, 158–159.

In the Columbia fragment, the length of the south side, as usual, is written on the left. The numeral νς refers to the area of the plot, 56 arourai. No other numerals are discernible, which suggests that in this codex the diagrams had only the givens indicated. For a similar drawing representing a plot of land in a mathematical exercise, cf. the diagram to problem O2 in P.Math. O recto, where, however, both the givens and the answer are marked.

P.Col. inv. 157a and a Peculiar Type of Mathematical Problems

The papyrus fragment comes from a page of a codex. It is broken off on top and bottom. The left margin on Side A (↓), which was the right-hand side page in the codex, is preserved in most of the upper two thirds of the fragment, and not much seems to be missing on the right since several lines are complete there. This means that the original page width likely did not exceed 12 cm and was probably closer to 11 cm. The codex thus belonged either to Turner's Group 8, if the height of the page was about twice the width, or Group 10, if the height did not exceed the width by very much (TURNER 1977, 20–22). Considerations of the content suggest that the latter was likelier than the former because not very many lines seem to be lost between the preserved text at the end of Side A (↓) and the text on Side B (→).

In the ed.pr., the papyrus is interpreted as containing three partially preserved problems, each of which asks to find the area of a quadrilateral plot of land from the given lengths of its sides. In fact, there are likely only two, not three problems, with the second beginning on Side A and continuing to Side B. Furthermore, the area is not the sought value, but the given in both problems. In no. 1, one side of the plot must also have been stated (cf. CBC no. 7), while in no. 2 only the area was. The task in both problems is to find the lengths of all the sides of the quadrilateral. The reason the nature of the problems was not recognized in the ed.pr. may lie in their strange—from the modern point of view—typology: the problems are indeterminate, that is, they have several or an infinite number of solutions, only one of which is obtained. That solution, however, is not randomly chosen, but is achieved by the same method in the two problems in this papyrus and in CBC no. 7 (cf. BAGNALL/JONES 2019, 35).

Finding one answer for a problem that has more than one solution must have been considered sufficient for solving the problem. This should not be surprising, since indeterminate problems are well attested in the papyrological evidence and outside it. For example, problem no. iv in P.Mich. III 145 col. III (2nd c. CE, provenance unknown) determines one set of numbers as a solution for a problem for which theoretically 24 sets can be found that satisfy the requirements of the problem's conditions; the so-called 100-birds problems, the earliest preserved attestation of which in the Mediterranean realm, widely understood, seems to be in Alcuin's *Propositiones ad acuendos juvenes*, tend to have several sets of solutions, of which only one is normally recorded (cf. FOLKERTS 1978, 27; HADLEY/SINGMASTER 1992, 106). Among problems transmitted in the Palatine Anthology in the form of epigrams, two have an infinite number of solutions, XIV 48 and 144, but since the epigrams never describe methods of solving the problems nor their answers, we have no means of knowing how they were envisaged to be solved, nor which solution was expected to be chosen. Nor did the scholiast, who supplied methods for solving many of the epigram-problems in the Palatine Codex (Paris suppl. grec 384), comment on these two epigrams.

The solving procedure used in the problems in the Columbia papyrus and in CBC no. 7 is based on factorization of the number with which the area of the plot is quantified. If this number contains a fractional part, it is first converted to smaller units so that the area is expressed in whole number terms. Then a pair of factors of this number is chosen, with the preference given either (a) to the pair in which one factor is the closest to the length of the given side (so in no. 1 here and in CBC no. 7), or (b) if no linear dimension is given, to the two highest factors that are closest to each other (no. 2). In the next step, the factor closest to the given linear dimension is doubled and the given side is subtracted from it; the result is assigned to the opposite side of the quadrilateral (no. 1, CBC no. 7). If no linear dimension is given, then the values of factors are assigned to the two pairs of opposite sides of the quadrilateral (no. 2). Finally, a demonstration is carried out, in which the area of the plot is computed upon the determined values of the four sides and found to equal the value of the area given in the statement of the problem.

Understanding the solving method of this type of problems and parallels from no. 2 in the Columbia papyrus and CBC no. 7 help us restore the lost part of no. 1, which was more then twice bigger than its preserved part. Even if we cannot be completely sure about the precise wording, the text of the entire problem must have been as follows: "[There is a plot of land measuring 3 1/4 1/8 schoinia on the south and with the area of 56 arourai. Find out the other sides. We do it this way. One schoinion contains eight hammata, so I resolve 3 1/4 1/8 schoinia into hammata: 3 1/4 1/8 times 8, it becomes 27. I resolve the arourai into hammata: 56 times 64, it becomes 3584. What multiplied times what is 3584? It is 32 times 112. 32 times 2, it becomes 64, subtract 27 from 64, the remainder is 37. Divide 37 by 8, it becomes 4 1/2 1/8. 4 1/2 1/8 on the north. Divide 112 by 8, it becomes 14.] // 14 on the east and 14 on the west. I have got a plot 3 1/4 1/8 schoinia on the south, 4 1/2 1/8 on the north, 14 on the east, 14 on the west 14. // To find out the area, we do it this way: I add the south and the north, 3 1/4 1/8 and 4 1/2 1/8, it becomes 8, half of which is 4; the west and the east, 14 and 14, half of which is 14. Multiply the halves by each other, it becomes 56 arourai".

ORCID®

Julia Lougovaya https://orcid.org/0000-0001-8099-5930

Bibliography

Sources and Sigla

CBC: Brashear, William (1990), "Mathematical School Exercise in Greek," in: *The Chester Beatty Codex Ac. 1390. Mathematical School Exercise in Greek and John 10:7–13:38 in Subachmimic* (Chester Beatty Monographs 13), Leuven/Paris, 33–56.

O.Bodl. II: *Greek Ostraca in the Bodleian Library at Oxford and Various Other Collections. Ostraca of the Roman and Byzantine Periods*, ed. by J. G. Tait and C. Préaux, London 1955.

P.Lond. II: *Greek Papyri in the British Museum* II, ed. by F. G. Kenyon, London 1898.

P.Math.: Bagnall/Jones 2019.

P.Mich. III: *Michigan Papyri* III, *Miscellaneous Papyri*, ed. by J. G. Winter et al., Ann Arbor 1936.

P.Tebt. I: *The Tebtunis Papyri* I, ed. by B. P. Grenfell, A. S. Hunt and J. G. Smyly, London 1902.

Cited Works

Bagnall, Roger S./Jones, Alexander (2019), *Mathematics, Metrology, and Model Contracts. A Codex from Late Antique Business Education* (P.Math.), New York.

Bakker, Marja (2007), "A Papyrus with Mathematical Problems," in: *Bulletin of the American Society of Papyrologists* 44, 7–21.

Folkerts, Menso (1978), *Die älteste mathematische Aufgabensammlung in lateinischer Sprache: Die Alcuin zugeschriebenen Propositiones ad Acuendos Iuvenes* (Österreichische Akademie der Wissenschaften Mathematisch-naturwissenschaftliche Klasse, Denkschriften 116/6), Vienna.

Fowler, David (1999), *The Mathematics of Plato's Academy. A New Reconstruction*, second edition, Oxford.

Friberg, Jöran (2005), *Unexpected Links between Egyptian and Babylonian Mathematics*, Hackensack (NJ).

Hadley, John/Singmaster, David (1992), "Problems to Sharpen the Young," in: *The Mathematical Gazette* 76 (475), 102–126.

Turner, Eric G. (1977), *The Typology of the Early Codex*, Philadelphia.

Figure Credits

Fig. 1, 2 P.Col. inv. 157a. Courtesy of The Rare Book & Manuscript Library, Columbia University. https://papyri.info/dclp/113807.