D OBSERVATIONS OF FLRW-DYNAMICS

In this chapter we should have a look at possible observations in FLRW-Universes in which the expansion velocity is proportional to the distance ($\upsilon = \mathrm{H}(t)r$), specifically how the Hubble-Lemaître constant H_0 can be determined, and how the dynamic evolution of the Hubble-function due to the gravitational interaction can be observed.

D.1 *Hubble-Lemaître constant* H_0

The Hubble-Lemaître constant H_0 can be determined in observations of Cepheid variable stars: Those stars have (i) a known relation between their pulsation period and their intrinsic brightness, and (ii) are bright enough to be seen in distant galaxies. Combining the estimate of the intrinsic brightness with the observed apparent brightness one can estimate the distance, which scales with H_0, or equivalently, h. Similar methods based on luminosity estimates of galaxies with the Tully-Fisher or Faber-Jackson relation are superseeded in their accuracy by Cepheids.

D.2 *Spatial curvature* Ω_K

By using the angular diameter distance of an object with known physical size, we can determine whether there is curvature in our universe, as this would influence the observed angular diameter. From observations of CMB-fluctuations or baryon acoustic oscillation features in the distribution of galaxies, for which very precise models exist and whose comoving distance is known, one can predict their angular size and compare to the measured angular size. Measurements point towards very small values for curvature, $\Omega_k < 0.01$.

D.3 *Supernova measurements and acceleration* q

By comparing the apparent luminosity with a prediction of the intrinsic luminosity (supernovae of type Ia are very suitable for this purpose, as the released energy is almost constant) and a measurement of redshift one can determine the evolution of luminosity distance d_L with redshift z or scale factor $a = 1/(1+z)$:

$$d_\mathrm{L} = ca\int_a^1 \frac{\mathrm{d}a}{a^2\mathrm{H}(a)} \tag{D.231}$$

for a spatially flat FLRW-cosmology. For a standard form of the Hubble-function

$$\mathrm{H} = \mathrm{H}_0 a^{-\frac{3(1+w)}{2}} \tag{D.232}$$

the above integral becomes divergent at the lower boundary if $w < -\frac{1}{3}$, corresponding to acceleration, and a supernova appears systematically darker. Typically, one would determine in for a model with two FLRW-fluids the matter density Ω_m and the equation of state w of the remaining dark energy fluid with density $1 - \Omega_m$, assuming a critical universe. The fact that supernovae appear systematically darker in accelerating universes is illustrated in Fig. 4, where models for the luminosity distance (and therefore, the distance modulus) for different Ω_m are compared to data.

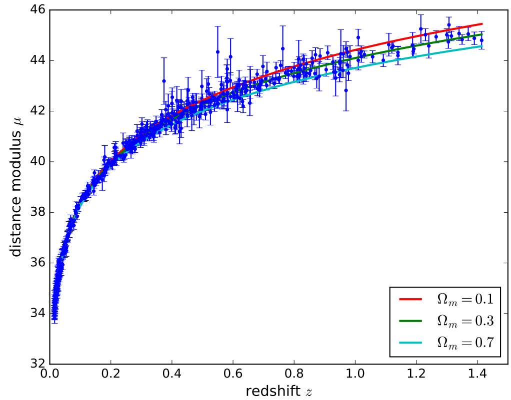

Figure 4: Supernova data and three different theoretical models for the distance modulus

The supernova data can be used to carry out a fit for Ω_m in a ΛCDM-cosmology, arriving at a value of $\Omega_m = 0.2785 \pm 0.013$, providing support for the existence of acceleration and the cosmological constant. The actual fit is shown in Fig. 5.

D.4 *(Finite) age of the Universe t_0*

The age of very old objects, for instance white dwarfs, one can put an lower bound on the age of the Universe,

$$t_0 = \int_0^1 \frac{\mathrm{d}a}{a\mathrm{H}} \tag{D.233}$$

which requires a period of decelerated expansion in the past to remain finite. Clearly, the magnitude of the integral is set by the inverse Hubble-Lemaître constant $1/\mathrm{H}_0$.

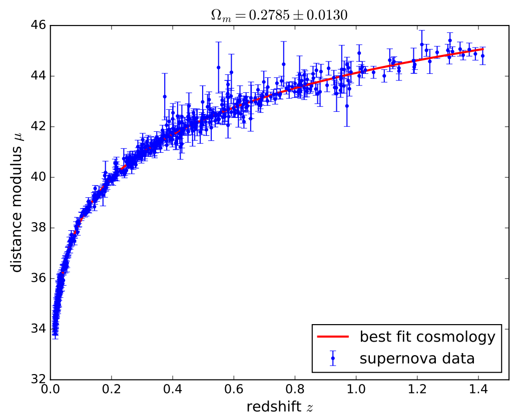

Figure 5: Supernova data and the best fitting ΛCDM-model