
CONTENTS

A Relativity and gravity	1
A.1 Why is classical Newtonian gravity insufficient?	1
A.2 What would be the most general classical theory of gravity?	2
A.3 Lorentz-geometry	6
A.3.1 Lie-groups and the generation of the Lorentz-group	9
A.4 Relativistic motion through spacetime	11
A.4.1 Variational principles for relativistic mechanics	11
A.4.2 Legendre-transforms and Hamilton-functions	13
A.4.3 Non-relativistic motion in weak gravitational potentials	16
A.4.4 Photon propagation on the Lorentzian spacetime	17
A.4.5 Photon propagation through weak gravitational fields	18
B Topological and metric structure of spacetime	21
B.1 Metric structure of manifolds and coordinate transforms	21
B.2 Locally Minkowskian structure and the equivalence principle	24
B.3 Vectors and fields on manifolds	24
B.4 Parallel transport and the covariant derivative	25
B.5 Geodesics as autoparallel curves	28
B.6 Geodesic motion through a variational principle	31
B.7 Equivalence and the relativistic origin of Newton's axioms	34
B.8 Geodesic deviation, curvature and gravity	36
C Differential structure of spacetime and curvature	39
C.1 Riemann curvature tensor	39
C.1.1 Riemann curvature in parallel transport	39
C.1.2 Riemann-curvature from covariant derivatives	40
C.1.3 What happens to vectors in parallel transport?	42
D Sources of the gravitational field	45
D.1 Gravity and matter	45
D.2 (Relativistic) fluids as sources of gravity	46
D.3 Fields as sources of gravity	50
E Gravitational field equation	53
E.1 What should be realised in a gravitational field equation?	53
E.2 Construction of the field equation	53
E.3 Ricci- and Weyl-curvature	55
E.4 Curvature invariants	56
E.5 Weak and static gravity	56
E.6 Weyl-curvature	57
E.7 Raychaudhuri-equation	60
E.8 Nonlinearity and locality	61
F Black holes	63
F.1 Schwarzschild black holes	63
F.2 Birkhoff's theorem	67
F.3 Conformal scaling of the Schwarzschild solution	67
F.4 Coordinate singularity at the Schwarzschild radius	69
F.5 Painlevé-Gullstrand-coordinates	70

F.6	Propagation of fields on a curved spacetime	71
F.7	Causal structure of black holes	74
F.8	Kruskal-coordinates	76
F.9	Reissner-Nordström black holes	77
F.10	Escape from a black hole	81
G	Friedmann-universes	83
G.1	Friedmann-Lemaître-Robertson-Walker cosmologies	83
G.2	FLRW-cosmologies as maximally symmetric spacetimes	84
G.3	Conformal flatness of FLRW-cosmologies	84
G.4	Spatial curvature of FLRW-cosmologies	85
G.5	Cosmological redshift	87
G.6	Cosmological horizons and causal structure	88
G.7	Friedmann-equations	88
G.8	Cosmological constant Λ	93
G.9	Size and age of FLRW-universes	93
G.10	Quintessence: dynamical fluids with varying w	94
H	Weak field gravity and gravitational waves	97
H.1	Weak field gravity and gravitational waves	97
H.2	Nonlinearities in the field equation	97
H.3	Gauging of the metric	98
H.4	Linearised gravitational field equation	99
H.5	Vacuum solutions of the linearised field equation	101
H.6	Stationary sources and gravitomagnetism	102
H.7	Wave equation and Lorenz-gauge condition	103
H.8	Plane gravitational waves in traceless transverse gauge	103
H.9	Huygens' principle and elementary waves	105
I	Gravity from a variational principle	109
I.1	Variational principles for particles and fields	109
I.2	Variational principles on manifolds	111
I.3	Gauge transformations on manifolds and source terms	112
I.4	Invariant volume elements	114
I.5	Einstein-Hilbert: gravity from a variational principle	114
I.6	Palatini-variation: metric $g_{\mu\nu}$ and connection $\Gamma_{\mu\nu}^\alpha$	117
I.7	Coupling to matter and generation of the energy momentum tensor	118
I.8	Dynamics of the energy-momentum tensor	119
I.9	Symmetries on manifolds: Lie-derivatives and the Killing equation . .	121
X	Mathematical supplement	125
X.1	Metric compatibility of the inverse metric	125